

Scattering amplitude and shift in self-energy in medium

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Abstract. Two simple proofs are presented for the first order virial expansion of the self-energy of a particle moving through a medium, characterised by temperature and/or chemical potential(s). One is based on the virial expansion of the self-energy operator itself, while the other is based on the analysis of its Feynman diagrams in configuration space.

1 Introduction

More than a decade ago, Leutwyler and Smilga [1] considered the problem of mass shift and damping rate of a nucleon propagating through a heat bath. As one of the methods to deal with the problem, they wrote (the first term of) the virial expansion of the nucleon self-energy, which relates it to the pion–nucleon scattering amplitude in the forward direction. At low pion density the first order formula yields the dominating contribution. The advantage of such a formula is that one can use the experimental data to compute the shift in self-energy without relying on any theoretical structure. Further the formula is simple enough to suggest a generalisation to other hadrons in different media. Accordingly a number of authors have used these relations to explore the properties of hadrons in such media [2–4].

In the context of quantum electrodynamics, Barton [5] took a physical approach to derive such formulae, following an earlier suggestion by Feynman [6]. The refractive index of a medium for a beam of particles of a definite frequency differs from unity due to the scattering of the incident beam by particles of the medium. The change in frequency (energy) of the particles may be related to this change in refractive index. In this way Feynman related the self-energy of the electron to its forward scattering amplitude with the virtual quanta in vacuum. Barton extended these considerations to the black body radiation.

Recently the complete expansion of the self-energy in the medium in powers of the distribution function has been proven in perturbation theory [7]. In effect, it is a re-ordering of the contributions in perturbation theory, which is an expansion in powers of coupling constant, to one in powers of the distribution function. The analysis is carried out in imaginary time formulation of field theory in the medium.

Here we present two simple proofs of the first term in the expansion of the self-energy. One is a generalisation of the method used in [8] and is based directly on the

virial expansion of the ensemble average of any operator. The other is in perturbation theory, based on Feynman diagrams in configuration space.

We shall work here in the real time formulation of field theory at finite temperature and density. Sections 2 and 3 describe the two proofs of the virial expansion. Section 4 discusses the relevance of such formulae and the possibility of extending the proofs to higher order.

2 Virial expansion

Following Weinberg [9], we introduce a compact notation for different types particles and their fields. Let $a(\vec{p}, \sigma, f)$ and $a^\dagger(\vec{p}, \sigma, f)$ be the destruction and creation operators for a particle of species f having momentum \vec{p} and spin projection σ ; then for example,

$$|\vec{p}, \sigma, f\rangle = a^\dagger(\vec{p}, \sigma, f)|0\rangle.$$

They satisfy the commutation (anticommutation) relations ($\omega_p = (\vec{p}^2 + m^2)^{1/2}$),

$$\begin{aligned} [a(\vec{p}, \sigma, f), a^\dagger(\vec{p}', \sigma', f')]_{\mp} \\ = (2\pi)^3 2\omega_p \delta^3(\vec{p} - \vec{p}') \delta_{\sigma\sigma'} \delta_{ff'}. \end{aligned}$$

The field operators are denoted by $\psi_l(x)$, where the index l denotes not only the field type but also runs over its components. Then $\psi_l(x)$ may be expanded as

$$\begin{aligned} \psi_l(x) = \sum_{\sigma} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} \left[u_l(\vec{p}, \sigma, f) a(\vec{p}, \sigma, f) e^{-ip \cdot x} \right. \\ \left. + v_l(\vec{p}, \sigma, f) a^\dagger(\vec{p}, \sigma, f^c) e^{ip \cdot x} \right], \end{aligned} \quad (2.1)$$

f^c denoting the antiparticle of the species f . The coefficient functions u_l and v_l depend on the spin of the particle. We have in mind the three types of fields, namely, the scalar field $\phi(x)$ for which $u_l = v_l = 1$; the Dirac

spinor field $\psi(x)$, for which they are the normalised Dirac spinors, $\bar{u}(\vec{p}, \sigma)u(\vec{p}, \sigma) = -\bar{v}(\vec{p}, \sigma)v(\vec{p}, \sigma) = 2m$ and the vector field A_μ , for which they are the polarisation vectors, $\epsilon_\mu(\vec{p}, \sigma), \epsilon_\mu^*(\vec{p}, \sigma)\epsilon^\mu(\vec{p}, \sigma') = -\delta_{\sigma\sigma'}$. Below we shall in most places suppress the variable σ of u_l, v_l, a and a^\dagger . If there is an integration over a 3-momentum, a summation over the corresponding σ as in (2.1) will also be implied.

In the real time formulation of field theory in medium, all Green functions, in particular the self-energy, acquire a $2 \otimes 2$ matrix structure. Since we confine here only to contributions linear in the distribution function, it will, however, suffice to consider only the 11-component of these matrices. Below we shall drop the 11 index¹.

Let us begin with the self-energy $\Sigma_f^{(0)}(p)$ of a particle of type f and momentum p in vacuum. For our purpose, we write this amputated, two-point Green function as

$$-i(2\pi)^4 \delta^4(p' - p) \bar{u}_l(\vec{p}, f) \Sigma_f^{(0)}(p) u_l(\vec{p}, f) = \langle 0 | a(\vec{p}', f) (S - 1) a^\dagger(\vec{p}, f) | 0 \rangle, \quad (2.2)$$

where the S -operator is given by the familiar time ordered expression,

$$S = T e^{i \int d^4x \mathcal{L}_I(x)},$$

$\mathcal{L}_I(x)$ being any interaction Lagrangian built out of the fields $\psi_l(x)$. Because $\Sigma_f^{(0)}(p)$ is one-particle irreducible by definition, it is understood here and below that we only retain such diagrams in matrix elements like the one on the right hand side of (2.2). The corresponding self-energy in the medium will be denoted by $\Sigma_f(p, u)$, where u^μ is the 4-velocity of the medium². Actually we shall work in the rest frame of the medium ($u^0 = 1, \vec{u} = 0$). Then we may write

$$-i(2\pi)^4 \delta^4(p' - p) \bar{u}_l(\vec{p}, f) \Sigma_f(p, u) u_l(\vec{p}, f) = \langle a(\vec{p}', f) (S - 1) a^\dagger(\vec{p}, f) \rangle, \quad (2.3)$$

where $\langle \dots \rangle$ denotes ensemble average: for any operator O ,

$$\langle O \rangle = \text{Tr}(\rho O), \quad \rho = e^{-\beta(H - \mu \mathcal{N})} / \text{Tr} e^{-\beta(H - \mu \mathcal{N})}. \quad (2.4)$$

Here H is the Hamiltonian, β^{-1} the temperature. For illustration we include a chemical potential μ for a fermionic species with the corresponding number operator \mathcal{N} .

The ensemble average $\langle O \rangle$ admits a virial expansion

$$\langle O \rangle = \langle 0 | O | 0 \rangle + \sum_{f'} \int \frac{d^3k}{(2\pi)^3 2\omega_k} n_{f'}(\omega_k) \langle \vec{k}, f' | O | \vec{k}, f' \rangle + \dots, \quad (2.5)$$

¹ The self-energy function Σ which actually shifts the pole position of the propagator is related to the components in a simple way [10]. With Σ_{11} these relations are $\text{Re}\Sigma(p) = \text{Re}\Sigma_{11}(p)$ and $\text{Im}\Sigma(p) = \alpha \text{Im}\Sigma_{11}(p)$, where $\alpha = \tanh(\beta p_0/2)$ for bosons and $\alpha = \coth\{\beta(p_0 - \mu)/2\}$ for fermions ($p_0 > 0$). Thus Σ_{11} deviates from Σ by exponential corrections at small temperatures. Further the factor α actually cancels on rearranging the distribution functions within the integral, at least for one-loop contributions [11]

² No confusion should arise from the use of same u in u^μ and $u_l(\vec{p}, f)$

where the sum over f' runs in general over the species of particles present in the medium. The dots represent terms of higher order in the distribution function n . The latter are given by the familiar expressions, $n_B(\omega_k) = (e^{\beta\omega_k} - 1)^{-1}$ for bosons and $n_F^\pm(\omega_k) = (e^{\beta(\omega_k \mp \mu)} + 1)^{-1}$ for fermions and antifermions respectively. Applying (2.5) to the right hand side of (2.3), we get for $\tilde{\Sigma}_f(p, u) \equiv \Sigma_f(p, u) - \Sigma_f^{(0)}(p)$,

$$-i(2\pi)^4 \delta^4(p' - p) \bar{u}_l(\vec{p}, f) \tilde{\Sigma}_f(p, u) u_l(\vec{p}, f) = \sum_{f'} \int \frac{d^3k}{(2\pi)^3 2\omega_k} n_{f'}(\omega_k) \langle \vec{k}, f' | a(\vec{p}', f) (S - 1) \times a^\dagger(\vec{p}, f) | \vec{k}, f' \rangle. \quad (2.6)$$

The matrix element on the right in this equation is recognised to be the amplitude T for scattering of a f -particle of momentum p with a f' -particle of momentum k in the forward direction,

$$\langle \vec{k}, f' | a(\vec{p}', f) (S - 1) a^\dagger(\vec{p}, f) | \vec{k}, f' \rangle = i(2\pi)^4 \delta^4(\vec{p}' - \vec{p}) T_{ff'}(p, k). \quad (2.7)$$

We thus get the virial expansion for the self-energy to first order,

$$-\bar{u}_l(\vec{p}, f) \tilde{\Sigma}_f(p, u) u_l(\vec{p}, f) = \sum_{f'} \int \frac{d^3k}{(2\pi)^3 2\omega_k} n_{f'}(\omega_k) T_{ff'}(p, k), \quad (2.8)$$

where an average over the polarizations of f -species and a sum over the polarizations of the different f' -species are understood.

We now apply this formula to two cases of interest. The first one is that of a nucleon in a medium. Its complete propagator in the medium is $i/(\not{p} - m - \tilde{\Sigma}_f)$. For the nucleon at rest ($\vec{p} = 0$) in the rest frame of the medium, we can write $\tilde{\Sigma}_f = a + b\gamma^0$, getting $\bar{u}(p) \tilde{\Sigma}_f u(p) = 2m_N(a + b)$. Also the complete propagator simplifies to

$$\frac{i}{p_0 - (m_N + a + b)} \cdot \frac{1}{2}(1 + \gamma^0).$$

Thus the new pole position is given by [1],

$$\tilde{m}_N - \frac{i}{2} \Gamma_N = m_N - \frac{1}{2m_N} \sum_{f'} \int \frac{d^3k}{(2\pi)^3 2\omega_k} n(\omega_k) T_{Nf'}(p, k). \quad (2.9)$$

The other case we consider is that of a vector meson in the medium. Let us rewrite (2.8) as

$$-\frac{1}{3} \sum_{\sigma} \epsilon_{\mu}^*(p, \sigma) \tilde{\Pi}^{\mu\nu}(p, u) \epsilon_{\nu}(p, \sigma) = \sum_{f'} \int \frac{d^3k}{(2\pi)^3 2\omega_k} n_{f'}(\omega_k) T_{Vf'}(p, k). \quad (2.10)$$

The free propagator for the vector meson is

$$D_{\mu\nu}^{(0)}(p) = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_V^2} \right) \frac{i}{p^2 - m_V^2}. \quad (2.11)$$

To sum the series of one-particle reducible insertions of the polarisation tensor, we have to decompose the latter in terms of kinematic covariants,

$$\tilde{\Pi}_{\mu\nu} = P_{\mu\nu}\Pi_t + Q_{\mu\nu}\Pi_l, \quad (2.12)$$

which we choose as

$$P_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} - \frac{p^2}{\bar{p}^2} \tilde{u}_\mu \tilde{u}_\nu, \quad Q_{\mu\nu} = \frac{p^4}{\bar{p}^2} \tilde{u}_\mu \tilde{u}_\nu, \quad (2.13)$$

where $\omega = u \cdot p$, $\bar{p} = (\omega^2 - p^2)^{1/2}$ and $\tilde{u}_\mu = u_\mu - \omega p_\mu / p^2$. The covariants are free from singularities at $p^2 = 0$, but at $\bar{p} = 0$, there is a constraint on the two amplitudes,

$$\Pi_t(p_0, \bar{p} = 0) = p_0^2 \Pi_l(p_0, \bar{p} = 0). \quad (2.14)$$

Then the full propagator becomes

$$D_{\mu\nu} = \frac{i}{p^2 - m_V^2 - \Pi_t} P_{\mu\nu} + \frac{i}{p^2 - m_V^2 - p^2 \Pi_l} \frac{Q_{\mu\nu}}{p^2} + \frac{i p_\mu p_\nu}{m_V^2 p^2}. \quad (2.15)$$

Thus in general the transverse and the longitudinal components suffer different shifts in the position of the pole. But for the vector meson at rest ($\bar{p} = 0$) the two shifts coincide, because of the constraint equation (2.14). We then get the same formula for the pole shift as (2.9) with the subscript N replaced by V .

3 Perturbation expansion

We now attempt a perturbative proof of (2.8). We begin by expanding the S -operator in (2.3) in the familiar perturbation series,

$$\begin{aligned} & -i(2\pi)^4 \delta^4(p' - p) \bar{u}_l(\vec{p}, f) \Sigma_f(p, u) u_l(\vec{p}, f) \\ &= \sum_{N=1}^{\infty} \frac{i^N}{N!} \int d^4 x_1 \cdots d^4 x_N F^{(N)}(x_1 \cdots x_N), \end{aligned} \quad (3.1)$$

where

$$\begin{aligned} & F^{(N)}(x_1 \cdots x_N) \\ &= \langle a(\vec{p}', f) T \{ \mathcal{L}_I(x_1) \cdots \mathcal{L}_I(x_N) \} a^\dagger(\vec{p}, f) \rangle_{\text{paired}}. \end{aligned} \quad (3.2)$$

The subscript ‘‘paired’’ indicates that $F^{(N)}$ is the sum of all connected terms obtained by pairing (contracting) all the operators in it in all possible ways. In other words, it represents the sum of all connected Feynman diagrams in the N th order. In the following we indicate a pairing by thick dots as superscript. The pairing between a creation or an annihilation operator of a particle with a field operator is given by

$$\begin{aligned} & a^\bullet(\vec{p}', f) \bar{\psi}_l^\bullet(x) = e^{ip' \cdot x} \bar{u}_l(\vec{p}', f), \\ & \bar{\psi}_l^\bullet(x) a^\dagger(\vec{p}, f) = e^{-ip \cdot x} u_l(\vec{p}, f). \end{aligned} \quad (3.3)$$

We may choose to work out the self-energy of a particle. But the medium may contain antiparticles. So we also note the contractions,

$$\begin{aligned} & a^\bullet(\vec{p}', f^c) \psi_l^\bullet(x) = e^{ip' \cdot x} v_l(\vec{p}', f), \\ & \bar{\psi}_l^\bullet(x) a^\dagger(\vec{p}, f^c) = e^{-ip \cdot x} \bar{v}_l(\vec{p}, f). \end{aligned} \quad (3.4)$$

Finally the pairing of two field operators results in the free propagator in the medium [12],

$$\psi_l^\bullet(x) \bar{\psi}_m^\bullet(y) = \langle T \psi_l(x) \bar{\psi}_m(y) \rangle. \quad (3.5)$$

A free propagator in the medium differs from that in vacuum if there are like-particles in the medium. In that case, it has an extra term containing the distribution function n of the particles in the medium and a mass-shell δ -function. Thus, isolating this term amounts to putting the internal line on mass-shell, i.e. opening the propagator into two external lines. When the δ -function is integrated out, this n -dependent piece in the propagator, to be denoted by a subscript n , becomes

$$\begin{aligned} & \langle T \psi_l(x) \bar{\psi}_m(y) \rangle_n \\ &= \int \frac{d^3 k}{(2\pi)^3 2\omega_k} \left\{ \pm n_f^+(\omega_k) e^{-ik \cdot (x-y)} u_l(\vec{k}, f) \bar{u}_m(\vec{k}, f) \right. \\ & \quad \left. + n_f^-(\omega_k) e^{ik \cdot (x-y)} v_l(\vec{k}, f) \bar{v}_m(\vec{k}, f) \right\}, \end{aligned} \quad (3.6)$$

where $+$ ($-$) sign before the first term holds if the species f is a boson (fermion). The distribution functions n^+ and n^- coincide if there is no chemical potential.

Let us assume, for simplicity, that only one species of particles has its free propagator altered in the medium by the additional term (3.6). We single out this field from the compact notation $\psi_l(x)$ and call it $\chi_l(x)$ and denote its species by f' ³. We now wish to collect all the linear contributions in n by opening each of the χ -propagators in turn in each of the Feynman diagrams for the self-energy in the N th order of its perturbation expansion (Fig. 1). To this end we consider the sum of Feynman diagrams, to be denoted by $F_{ij}^{(N)}(x_1, \cdots, x_N)$, containing a χ -propagator between any two vertices, say at x_i and x_j ,

$$\begin{aligned} & F_{ij}^{(N)}(x_1, \cdots, x_N) \\ &= \langle a(\vec{p}' f) T \{ \mathcal{L}_I(x_1) \cdots \mathcal{L}_I^\bullet(x_i) \cdots \mathcal{L}_I^\bullet(x_j) \\ & \quad \cdots \mathcal{L}_I(x_N) \} a^\dagger(\vec{p}, f) \rangle_{\text{paired}}, \end{aligned} \quad (3.7)$$

where we explicitly indicate the pairing of the two fields at x_i and x_j .

Consider first the indicated pairing in (3.7) before any other pairings are carried out, i.e., keeping all other operators in their respective positions in the T -product. By a certain number of interchanges of the field operators we bring them together to form $\langle \chi_l(x_i) \bar{\chi}_m(x_j) \rangle$. Then we extract its n -dependent contribution given by (3.6)⁴. Since

³ For self-conjugate species like π^0 , the two terms in (3.6) contribute to the same amplitude, the direct and crossed processes being identical

⁴ If f' is a fermionic species, there may be two such propagators between x_i and x_j . We, of course, have to extract this contribution from both of them

we restrict ourselves to linear contributions in n , we set all other in-medium propagators to their vacuum values.

In order to interpret the resulting expression in terms of a two-body scattering amplitude, we use (3.3) to rewrite the coefficient functions for the f' species in $\langle \chi_l(x_i) \bar{\chi}_m(x_j) \rangle$ as pairings,

$$\begin{aligned} & e^{-ik \cdot (x_i - x_j)} u_l(\vec{k}, f') \bar{u}_m(\vec{k}, f') \\ &= \chi_l^\bullet(x_i) a^{\dagger \bullet}(\vec{k}, f') a^{\bullet \bullet}(\vec{k}, f') \bar{\chi}_m^{\bullet \bullet}(x_j). \end{aligned} \quad (3.8)$$

For the antiparticle species f'^c , we use (3.4) to write an analogous equation,

$$\begin{aligned} & e^{ik \cdot (x_i - x_j)} v_l(\vec{k}, f') \bar{v}_m(\vec{k}, f') \\ &= a^{\bullet}(\vec{k}, f'^c) \chi_l^\bullet(x_i) \chi_m^{\bullet \bullet}(x_j) a^{\dagger \bullet \bullet}(\vec{k}, f'^c). \end{aligned} \quad (3.9)$$

Now the χ fields can be brought back to their original positions forming again the complete vertices $\mathcal{L}_I(x_i)$ and $\mathcal{L}_I(x_j)$. We also bring the creation and the annihilation operators respectively to the right and the left of the T -product getting

$$\begin{aligned} & F_{ij,n}^{(N)}(x_1, \dots, x_N) \\ &= \int \frac{d^3k}{(2\pi)^3 2\omega_k} n(\omega_k) \langle 0 | a(\vec{p}', f) a^{\bullet \bullet}(\vec{k}, f') T \{ \mathcal{L}_I(x_1) \\ & \quad \cdots \mathcal{L}_I^\bullet(x_i) \cdots \mathcal{L}_I^{\bullet \bullet}(x_j) \cdots \mathcal{L}_I(x_N) \\ & \quad \times a^{\dagger \bullet}(\vec{k}, f') a^{\dagger}(\vec{p}, f) | 0 \rangle_{\text{paired}}, \end{aligned} \quad (3.10)$$

where the subscript n denotes the n -dependent contribution from the χ -propagator connecting x_i and x_j .

It remains to show that we do not get any additional sign, if the χ field is fermionic. First note that bringing the fields $\chi(x_i)$ and $\bar{\chi}(x_j)$ together to form the propagator and then putting them back to their old positions can be effected by the same set of field interchanges, one in reverse order of the other. Thus we do not encounter any extra minus sign here. Also the interaction Lagrangians being quadratic in fermionic χ , the operators $a(\vec{k}, f')$ and $a^\dagger(\vec{k}, f')$ do not produce any minus sign while moving through them. However, in the left hand side of (3.8) an initial interchange of a and a^\dagger is needed which produces a minus sign to cancel the minus sign in front of the first term in (3.6). Thus (3.10) remains valid for both fermionic and bosonic operators.

Equation (3.10) gives the linear contribution from a definite χ -propagator, namely the one between x_i and x_j . To get the total contribution from all the χ -propagators in the diagrams, we must allow $a(\vec{k}, f')$ and $a^\dagger(\vec{k}, f')$ to be paired with χ fields at all vertices in all possible ways, not just with $\chi(x_i)$ and $\bar{\chi}(x_j)$. Thus we get

$$\begin{aligned} & F_n^{(N)}(x_1, \dots, x_N) \\ &= \int \frac{d^3k}{(2\pi)^3 2\omega_q} n(\omega_q) \langle 0 | a(\vec{p}', f) a(\vec{k}, f') \\ & \quad \times T \{ \mathcal{L}_I(x_1) \cdots \mathcal{L}_I(x_N) \} a^\dagger(\vec{k}, f') a^\dagger(\vec{p}, f) | 0 \rangle_{\text{paired}}. \end{aligned} \quad (3.11)$$

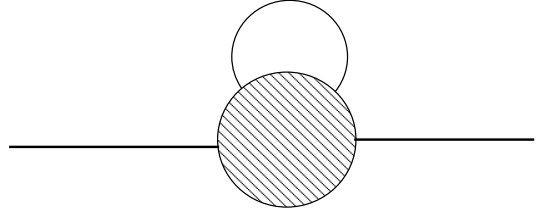


Fig. 1. The virial expansion of self-energy in medium to first order

This matrix element is just the sum of all Feynman diagrams in coordinate space in the N th order of perturbation expansion for the scattering amplitude $T_{ff'}$ introduced previously by (2.7). We thus prove (2.8) in an arbitrary order of perturbation theory.

4 Discussion

It is well known that the effective theory incorporating the symmetries of the QCD Lagrangian, called the chiral perturbation theory, can successfully describe the strong interaction processes in the low energy region [13]. This theory finds a natural application in the realm of hadronic statistical physics [14–16]. There is also a phenomenological approach in statistical mechanics for interacting systems, namely the method of virial expansion.

In a region where the expansions of both the methods converge rapidly, the virial expansion would prove to be an identity in chiral perturbation theory. However, there are situations, due to the proximity of resonances, for example, where the effective coupling constants in the chiral Lagrangian can be rather large and the leading term in the chiral expansion may hold only in a limited region of interest. The virial expansion, on the other hand, may enjoy a wider range of validity.

The case of the nucleon self-energy at finite temperature illustrates this point [1]. Due to the presence of the $\Delta(1232)$ resonance near the πN threshold, the chiral expansion converges slowly, whereas the first term in the virial expansion is a good representation for low enough pion densities. Another example is the nucleon self-energy in a nuclear medium, which involves the interaction of the two-nucleon system. Here the presence of bound or virtual two-nucleon states close to the threshold of NN scattering makes it difficult to formulate a satisfactory chiral perturbation theory for this system [17]. But the virial formula is expected to hold for densities close to the nuclear saturation density [18].

The proofs for the virial expansion described here are indeed simple, mainly because we restrict to the first order formula. But the methods are not restricted to the first term in any way. They may well provide simple alternative proofs in the real time formalism for the complete virial expansion [7]. One has only to take care of two additional aspects, namely, the disconnected parts that result from opening more than one internal lines and the $2 \otimes 2$ matrix structure due to the so-called ghost vertices.

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